# Predictive Functionnal Control of a Sendzimir cold rolling mill

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Abstract- This paper presents the implemtation of a predictive functional controller for the sendzimir 20-high cold rolling mill. The complete mill model is presented and contains two parts, the static part modelled by a gain matrix, and the dynamic part which represented by a third order transfer function with a pure time delay. The PFC controller formulation is derived using an embedded model in an ARIMAX form, this is done in order to take account of the pure time delay between measurement and actuation. The PFC control algorithm need less calculation time and has a set control parameters which can be easily tuned. Simulation studies are carried out by a comparison with benchmark PID controller. Results shows that the PFC approach provides a better control performances, quicker setpoint reaching and disturbances (shape fluctuations) overcoming.

 $\mathit{Keywords-}$  Sendzimir steel mills, Predictive functional control, Shape control

#### I. INTRODUCTION

Model predictive control (MPC) refers to a wide family of control algorithms that employ an embedded model to predict the future behavior of the process over an extended prediction horizon. These algorithms are formulated as a performance objective function, which is defined as a combination of setpoint tracking performance and control effort. The first controller output move is implemented, and then the entire procedure is repeated at the next sampling instance. The goal behind the emergence of using MPC, and PFC in particular, is to reduce occurring variance in the Controlled Variable (CV), This may allow lowering of the control setpoint target or improving disturbance rejection, therefore reducing cost and improving energy savings. MPC was found to be very effective in tackling such control problems, due to the ability of prediction, given by the embedded model, as well as effective constraint handling[6]. MPC was successfully used in shape control problems especially in the paper domain [9]. However, MPC has so far and at the best knowledge of the future never been implemented for a Sendzimir steel mill. The paper present an application of a class of model predictive control represented here by PFC. In sections II. and III., a comprehensive description and a mathematical model of the Sendzimir steel mill is presented. Section IV. details the steps taken to derive a high order PFC formulation, as well as a broad overview on PFC tuning. Section V. details the design and results of a higher order PFC for the shape control of the model given in section III. and implemented following the control law obtained in section IV.. In section VI., a benchmark PID is designed and control performances are assessed. Finally Section VII., conclude with a comprehensive comparison between PFC and PID control performances in terms of Mean Square Error (MSE), response time and overshooting.

#### II. SENDZIMIR MILL DESCRIPTION

The Sendzimir mill is a reversing mill, and a separate schedule containing a number of passes is specified for each different material rolled. One pass schedule can contain from 4 to 15 passes through the rolling cluster. Each pass involves different entry and exit gauges, with minor changes in the material hardness from pass to pass. The mill under consideration is the 20-high mill, with the rolls arranged in a 1-2-3-4 configuration, above and below the strip (see Fig. 1). This configuration is used for rolling hard materials such as stainless steel, the large stack providing support and preventing unwanted bending under the high loads involved. Hydraulic motors are used to drive the cluster via the second intermediate rolls. The first inter-mediate rolls (FIRs) are tapered in opposite directions, their lateral movement affording one means of shape control. The other method of shape control is via the segmented back-up rolls at the top of the mill. Movement of the As-U-Roll (AUR) racks in or out of the mill cause rotation of eccentrics on the top mill back-up roll shafts which create bending of the back-up roll, this bending permeates through the cluster and is attenuated due to the stack rigidity, the stack behaving like a low-pass filter, in spatial terms. Due to the closer proximity of the FIRs to the strip, their influence as a shape control device is considerable. Both sets of shape actuators are driven by hydraulic motors, which operate at a single speed only. The high-order bending, which is achievable in the Sendzimir mill, allows correction of high-order shape defects such as herringbone and quarterbuckle.



Fig. 1. The Sendzimir cold rolling mill

The difficulty of shape control, being a truly multivariable control problem, is manifested by the fact that shape, in the vast majority of multi-roll mills, was and in most cases still controlled using manual control actions [7]. Although there are a variety of mechanisms for controlling strip shape, the unique method employed in the Sendzimir mill is eccentric position control, which gives considerable variety in the types of roll bending which can be achieved. Such variety and shape control potential results in a challenging control problem.

#### III. MODEL OF THE MILL

The mill model is represented by two parts.

- · The static model which represents the rolling cluster.
- Dynamic model deals the effects of hydraulic actuators, strip dynamics and the shapemeter.

#### A. Static model

Since the rolling cluster is under a high compressive load (approx. 5000 tonnes), changes in shape actuator positions are transmitted immediately to the roll gap. Therefore, the relationship between the shape actuator positions and the roll-gap shape profile is represented by a matrix of constant gains (the mill matrix) as:

 $G_m = [G_a \ G_i] \in \mathfrak{R}^{8 \times 10}$ 

And:

$$y_g = G_m \cdot \begin{bmatrix} u_a \\ u_i \end{bmatrix} \in \mathfrak{R}^d$$

Represents the shape profile at the roll gap, where  $u_a \in \Re^8$  and  $u_i \in \Re^2$  are respectively the As-U-Roll (AUR) and First Intermediate Roll (FIR) actuator positions. The adoption of a set of linear gains carries with it the assumption of the theorem of superposition i.e. the net shape effect at the roll-gap is equal to the sum of the individual effects due to AUR and FIR movements separately.

TABLE I PARAMETERS OF AUR AND FIR ACTUATOR

Actuator	ð	$\tau_c$	k <sub>i</sub>	$\Delta$	$k_i$
Units	volts	Sec	Gain	mm	Gain
AUR	0.25	0.05	8.0	0.15	1.3
FIR	0.25	0.1	3.13	0.25	0.7

## B. Dynamic model

Both Actuators are represented by the block diagram 2, it contains a significant nonlinearity which is an obstacle to the diagonalisation of the system transfer function matrix, a linearisation technique must be used.



Fig. 2. Block diagram of the actuator subsystem

The features of the actuator subsystem includes different parameters for both AUR and FIR, rate-limited movements, backlash in drive mechanisms, these parameters are given in Table I as follows: The strip and shape meter dynamics vary with the rolling speed and

$$g_{ss}(s) = \frac{e^{-\tau_1 s}}{(1 + \tau_2 s)(1 + \tau_3 s)} \tag{1}$$

where:  $\tau_1 = D/v$ ;  $\tau_2 = D_1/v$ 

D: distance from roll-gap to shapemeter (2.91 m).

 $D_1$ : distance from roll-gap to coiler (5.32 m).

v: Rolling speed.

are given by:

 $\tau_3$  is the time constant of the shapemeter filter, which varies as follows: The strip dynamics relate to the transport delay between actua-

TABLE II VARIATION OF THE SHAPEMETER TIME CONSTANT

Speed (m/s)	$0 \rightarrow 2$	$2 \rightarrow 5$	$5 \rightarrow 15$
$\tau_3$ (Sec.)	1.43	0.74	0.3

tion and measurement and the principle of St. Venant (Bryant, 1973), which states that the stress variation caused by end traction will decay to zero exponentially due to the difference between input and exit sides of the mill.

#### **B.1-** Actuator linearisation

A simple describing function analysis [7] is used to represent actuator subsystem in Fig.2 as:

$$g_{act}(s) = \frac{1}{1 + \left(\frac{\pi x}{4k_c k_i}\right) (1 - \delta^2 / x^2)^{\frac{1}{2}s}}$$
(2)

where x is the signal entering the relay. A first-order compensator is now placed in cascade with each actuator of the form:

$$C(s) = \frac{1 + \tau s}{1 + \tau_e s} \tag{3}$$

with  $\tau$  is evaluated as the time constant in 2 and  $\tau_e$  chosen by the designer, subject to limitations on the max. rate of change of the actuator positions. A value of  $\tau_e = 2$  was found to be appropriate.

#### C. Complete mill model

Following linearisation of the actuators, the complete mill model may now be stated as:

$$G(s) = g(s) [G_a \ G_i] \in \mathfrak{R}^{8 \times 10}$$

$$\tag{4}$$

where:

$$g(s) = \frac{e^{-\tau_1 s}}{(1 + \tau_2 s)(1 + \tau_3 s)(1 + \tau_e s)} = \frac{\gamma(s)}{\sigma(s)}$$
(5)

Features of the mill model which present a challenge to the control engineer include the following:

- The mill matrix,  $G_m = [G_a \ G_i]$ , suffers from rank deficiency, there being only four reasonably large singular values.
- The mill matrix,  $G_m$ , varies with each schedule and pass, as the mill set-up and strip parameters change.
- The dynamic section of the system, represented by g(s) varies with mill speed.
- The complete system, including the FIRs, is nonsquare, preventing an attempt at associating particular inputs and outputs. However, with the AUR system alone, such an association is possible by relating actuators and measurements in the same region of the strip.

# IV. PREDICTIVE FUNCTIONAL CONTROL FORMULATION

All MPC strategies use the same basic approach *i.e.*, prediction of the future plant outputs, and calculation of the manipulated variable for an optimal control [2],[9]. Most MPC strategies are based on the following principles [5]:

- Use of an internal model: its formulation is not restricted to a particular form, and the internal model can be linear, nonlinear, state space form, transfer function form, first principles, blackbox etc. In PFC, only independent models where the model output is computed only with the present and past inputs of the process models are used.
- **Specification of a reference trajectory:** From the measured output value of the plant, a future desired trajectory is defined. We do not aim at the set point, whatever constant or time varying, but at a trajectory leading to the set point in a specific way. Usually for PFC, the reference trajectory is an exponential.
- **Determination of the control law:** The control law is derived from the minimisation of the error between the predicted output and the reference with, in PFC, the projection of the Manipulated Variable (MV) on a basis of functions [3]. It is claimed in [3], that it is more efficient to structure the manipulated variable that way as: on one hand, we limit the number of unknown parameters  $N_b$  by projecting the future manipulated variable onto a base of functions  $UB_j$  of smaller dimension than the prediction horizon H. On the other hand, the discontinuity, and the control frequency range, are limited, by limiting the dimension of the basis.

The future command is then of the form:

$$u(k+i) = \sum_{j=1}^{N_b} u_j(k) U B_j(i)$$
(6)

where the  $UB_i$  are the basis functions.

Every input base  $UB_j$  implies an output basis  $SB_j$  known *a priori* for a given model. For example if we take a polynomial base, the first three basis functions are shown in Figure 6. The concept of projecting the MV onto a functional basis can be found in [3][4] and is given the name Predictive **Functional** Control (PFC). Usually a zero order base function  $(UB_0)$ , representing a step MV change to find at each sample time, is used.

Therefore based on the above principles, the PFC algorithm may be of several levels of complexity depending on the order and form of the internal model, the order of the basis function used to decompose the MV and the reference trajectory.

#### A. First order PFC

If the system can be modeled by a first order plus pure time delay, then the following steps in the development of the control law are taken.

#### A.1- Model formulation

A typical first order transfer function equation (7) in order to implement a basic first order PFC is used.

$$y_M(\mathbf{s}) = \frac{K_M}{1 + \tau_M \mathbf{s}} u(\mathbf{s}) \tag{7}$$

Note that the time delay is not considered in the internal model formulation and in this case  $K_M$  is equal to one. The discrete time formulation of the model zero-order hold equivalent, is then obtained in (8)

$$y_M(k) = \alpha y_M(k-1) + K_M(1-\alpha)u(k-1)$$
 (8)

where  $\alpha = e^{\left(-\frac{I_s}{\tau_M}\right)}$ . If the manipulated variable is structured as a step basis function:

$$y_L(k+H) = \alpha^H y_M(k) \tag{9}$$

$$y_F(k+H) = K_M(1-\alpha^H)u(k) \tag{10}$$

Where,  $y_L$  and  $y_F$  are, respectively, the free (auto regressive) and the forced response of  $y_M$ .

#### A.2- Reference trajectory formulation

If  $y_R$  is the expression of the reference trajectory, then at the coincidence point *H*:

$$C(k+H) - y_R(k+H) = \lambda^H (C(k) - y_P(k))$$
(11)

with  $\lambda = e^{\left(-\frac{T_S}{T_R}\right)}$ , where:  $T_S$  is the sampling period and  $T_R$  is the Closed Loop Response Time (CLRT) to be specified. thus:

$$y_R(k+H) = C(k) - \lambda^H (C(k) - y_P(k))$$
 (12)

A.3- Predicted process output

The predicted process output is given by the model response, plus a term given the error between the same model output and the process output:

$$\hat{y}_P(k+H) = y_M(k+H) + (y_P(k) - y_M(k))$$
 (13)

A.4- Computation of the control law

At the coincidence point *H*:

$$y_R(k+H) = \hat{y}_P(k+H) \tag{14}$$

Using equations (9), (10), (12) and (13) we obtain:

$$C(k) - \lambda^{H}(C(k) - y_{P}(k)) - y_{P}(k) = y_{M}(k + H) - y_{M}(k)$$
(15)

Replacing  $y_M(k+H)$  by its equivalent in equations (9) and (10) we obtain:

$$C(k)(1 - \lambda^{H}) - y_{P}(k)(1 - \lambda^{H}) + y_{M}(k)(1 - \alpha^{H})$$
  
=  $K_{M}(1 - \alpha^{H})u(k)$  (16)

Solving for u(k), the final result is the control law given in (17).

$$u(k) = \frac{(C(k) - y_P(k))(1 - \lambda^H)}{K_M (1 - \alpha^H)} + \frac{y_M(k)}{K_M}$$
(17)

Note that any measured disturbance may be compensated in a feedforward manner from equation (17).

# B. Higher order PFC

For higher order system representations, a simplified PFC may be given in [4], and developed in what follows:

### B.1- Model formulation

If we consider a given model, of a specific process, given on the following Auto-Regressive with Exogenous inputs form:

$$y_{M}(z) = \frac{b_{0} + b_{1}z^{-1} + b_{2}z^{-2} + \dots + b_{m}z^{-m}}{1 + a_{1}z^{-1} + a_{2}z^{-2} + \dots + a_{n}z^{-n}}u(z) + \frac{c_{0} + c_{1}z^{-1} + c_{2}z^{-2} + \dots + c_{q}z^{-m}}{1 + a_{1}z^{-1} + a_{2}z^{-2} + \dots + a_{n}z^{-n}}v(z)$$
(18)

leading to the following difference equation:

$$y_M(k) = -\sum_{i=1}^n a_i y_M(k-i) + \sum_{i=0}^m b_i u(k-i) + \sum_{i=0}^q c_i v(k-i)$$
(19)

where u is the MV and v is the input disturbance. Any complex poles will be induced in the values of the denominator values  $a_n$ .

The model response, excluding the input disturbance v(k) may be decomposed into two distinctive parts: a free (autoregressive) and a forced responses.

The free (autoregressive) response of the system is given by  $y_A$ , when u(k) = v(k) = 0, as:

$$y_A(k) = -\sum_{i=1}^n a_i y_A(k-i)$$
(20)

The future free response of a linear system, at k + 1, can also be given using the future *n* elementary outputs  $y_1(k+1)$ ,  $y_2(k+1)$  to  $y_n(k+1)$  as:

$$y_A(k+1) = y_1(k+1)y_M(k) + y_2(k+1)y_M(k-1) + \dots + y_N(k+1)y_M(k-N+1)$$
(21)

where  $y_i(k+1)$  are obtained from equation (19) setting all initial conditions to zero except one set to unity successively, as:

In practice, we require to iterate the elementary outputs  $y_i(k+H) n$ times, starting at time k = 1. This is done only once at the start of the real time calculations of the MV, with all initial conditions null except one until reaching the coincidence point *H*, thus obtaining a set of values  $y_i(k+h)$ . These values are stored in a memory and used to obtain the free response, equation (22) at the coincidence point *H*.  $y_A(k+H) = y_1(k+H)y_M(k) + y_2(k+H)y_M(k-1)$ 

$$+\cdots+y_n(k+H)y(k-n+1)$$
(22)

The calculation of the elementary outputs, are comparable to the calculation of the prediction model parameters in Generalized Predictive Control (GPC) [1]. The difference lies in the nature of the internal model formulation, an independent auto regressive one for PFC. The elementary outputs obtained, are of a vector form, due to the use of a single coincidence point H. If a coincidence horizon is considered, the elementary outputs will form a matrix as in GPC.

Note that the formulation of the free response, as in equation (21), is only possible when using independent models as all future and past value of y(k) are known, a unique attribute of the independent model formulation [8].

The forced response is calculated going from null initial conditions and applying a step input, and is given by:

$$Y_F(k+H) = \sum_{i=0}^{m} b_i u(k-i+H)$$
(23)

*B.2- Computation of the control law* At the coincidence point *H*:

$$y_R(k+H) = \hat{y}_P(k+H) \tag{24}$$

Using equations (22), (23), (12) and (13) we obtain:

$$C(k) - \lambda^{H}(C(k) - y_{P}(k)) - y_{P}(k)$$
  
=  $y_{M}(k+H) - y_{M}(k)$  (25)

Replacing  $y_M(k+H)$  by its equivalent in equations (22) and (23) we obtain the higher order PFC following control law.

$$u(k) = \frac{(C(k) - y_P(k))(1 - \lambda^H) + y_M(k)}{K(1 - \sum_{i=1}^n p_i)} - \frac{\sum_{i=1}^n p_i y_M(k - i)}{K(1 - \sum_{i=1}^n p_i)}$$
(26)

where  $p_1 = y_1(k+H)$ ,  $p_2 = y_2(k+H)$ ,  $\cdots p_N = y_n(k+H)$  and *K* is the overall gain of the system.

In order to compensate the effect of a measured disturbance v(k), a solution is to eliminate its effect in a feedforward manner as shown in equation (27).

$$u(k) = \frac{(C(k) - y_P(k))(1 - \lambda^{I'}) + y_M(k)}{K(1 - \sum_{i=1}^n p_i)} - \frac{\sum_{i=1}^n p_i y_M(k - i)}{K(1 - \sum_{i=1}^n p_i)} - \frac{\sum_{i=0}^q c_i v(k - i)}{K_d(1 - \sum_{i=1}^n p_i)}$$
(27)

where  $K_d$  is the disturbance overall gain.

If the internal model is of first order, i.e., n=1, then the control law equation (27) becomes the control obtained for a first order internal model equation (17).

Some guidance in the determination of the co-incidence horizon can be provided as:

- **Complex poles:** Choose *H* to be, at least, one period of the open-loop complex response, and
- Non-minimum phase: Longer than the inverse 'dip' in the step response.

#### C. Case of a process with a pure time delay

In the linear case, a process with a pure time delay can be expressed in terms of a delay-free part, plus a delay added at the output, as in Fig. 3. The value  $y_{P_{delay}}$  at time k is measured, but not  $y_P$ . In order to take into account the delay in a control law formulation, prior knowledge of the delay value d is needed.  $y_P$  can be estimated as:

$$y_P(k) = y_{P_{delay}}(k) + y_M(k) - y_M(k-d)$$
(28)



Fig. 3. Process with time delay

and replaced in equations (17), (27) and (26).

#### D. Tuning in PFC

According to the three principles of PFC (Section IV.), tuning is a function of the order of the basis constructing the MV, the reference trajectory, the control horizon and the CLRT value.

A general idea of the influence of PFC parameters, on precision, transient response and robustness are graded between 0 (indicating minimum influence) and 100 (indicating maximum influence), is given in Table III.

 TABLE III

 EFFECT OF PFC PARAMETERS ON RESPONSE AND ROBUSTNESS:

 TUNING IN PFC

	SS Resp.	Transient Resp.	Robus- tness
Basis Function	100	0	0
Reference Trajectory	0	100	50
Coincidence horizon	0	50	100

In most cases, an exponential reference trajectory is chosen along with a single coincidence horizon point (H = 1) and a zero order basis function [3] *i.e.* a step change to find. Considering the known Open Loop Response Time of the system (OLRT), one can choose the CLRT value given by the ratio OLRT/CLRT. This ratio then becomes the major tuning parameter shaping the system output and MV, dictating how much overshoot occurs and ensuring stability, on the condition that the internal model is accurate enough. For slow processes, *e.g.*, heat exchange systems, a ratio of 4 or 5 is found most suitable, and ensures a stable PFC [4].

#### V. CONTROLLER IMPLEMENTATION AND RESULTS

Results of  $3^{rd}$  order PFC controller implementation for automatic shape control will be presented, only AUR actuators are used here as a shape control device. The basis for not including (initially, at least) the FIRs in the automatic control scheme accords with rolling practice, since the FIRs are generally preset for a particular pass, with only the AURs (as the name suggests) moved while rolling is taking place[6]. The model is decoupled by using an inverse of  $G_a \in \Re^{8\times 8}$ as a precompensator, the resulting complete mill model contains 8 independent SISO tranfer functions g(s) with a time delay fixed to  $\tau_1 = 0.728sec$ .

The performance of the implemented controller could be seen in 4, each of the 8 segements is controlled separately and reaches the setpoint value quickly (in 10 sec.), to illustrate the ability of controller



to overcome disturbances, a step change of the incoming shape pro-

file simulating a weld in the strip is applied at t = 30 sec., the main

feature of this controller is the non agressive control signals.

Fig. 4. Segments evolution for high order PFC



Fig. 5. Shape profile evolution for high order PFC

# VI. BENCHMARK PID DESIGN

In this section a MIMO PID controller will be designed and tuned, as best as we could, to regulate the shape profile. The PID controller transfer function is usually given by:

$$C(s) = K_p + \frac{K_i}{s} + K_d s \tag{29}$$

where Kp, Ki and Kd are, respectively, the proportional, integral and derivative gains. A digital version of the classical PID is used when the plant is operated by any digital/computer based controller, and can be given by the following set of equations, assuming numerical integration derivative approximations obtained using the backward difference:

$$e(k) = y_{ref}(k) - y(k) \tag{30}$$

$$s(k) = s(k-1) + e(k)$$
 (31)

And the digital PID control law is given by:  

$$u(k) = K_p \{e(k) + K_i s(k) + K_d [e(k) - e(k-1)]\}$$
(32)

# where : $K_i = \frac{T_s}{T_i} K_d = \frac{T_d}{T_s}$ , and $T_s$ is the sampling time.

Since PID has not the power of prediction, performances reached with this controller are not good as ones obtained with the PFC technique (see Fig. 6). Despite that, and the presence of time delay, PID controller can reject disturbances in the incoming shape profile, but with a slower dynamic.



Fig. 6. Segments evolution for PID controller



Fig. 7. Shape profile evolution for PID controller

TABLE IV COMPARISON BETWEEN CONTROL PERFORMANCES

	Response	Overshoot	Mean Square
	time (Sec.)	(%)	Errors $(N/mm^2)$
PFC	13.1	0	17.17
PID	21	1.85	26.92

# VII. CONCLUSION

From the results obtained in section V. and VI., summarized in table VI., it is clear that PFC overcome the performances of PID controller not only takes into account the pure time delay of the system, but giving the power of prediction reduces the aggressive nature of the

control action, resulting in a smoother and quicker response with a good overcoming of shape fluctuations in the rolled strip by using an exact model of the plant, the robusteness to the model-plant mismath was therefore increased which ensures a best control performances.

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